1. Algebra

Momentum Learning

1.1 Square Roots

1.1.1 Rationalizing The Denominator

Changing the denominator of a fraction from an irrational number to a rational number is called ***rationalizing the denominator*** of the fraction. Rationalizing the denominator does not change the value of the original fraction. To rationalize the denominator, we multiply the numerator and denominator of the fraction by a suitable factor. For example, if the fraction is in the form $\frac{\sqrt{a}}{\sqrt{b}}$, we multiply both the numerator and the denominator by $\sqrt{b}$

So, $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{a \cdot b}}{\sqrt{b \cdot b}} = \frac{\sqrt{ab}}{b}$, Note that $\frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{ab}}{b}$ have the same value: they are equivalent fractions.

Look at some more examples:

- $\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{3\cdot 2}}{\sqrt{2\cdot 2}} = \frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$
- $\frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$, and
- $\frac{3\sqrt{5}}{2\sqrt{2}} = \frac{3\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\cdot\sqrt{5\cdot 2}}{2\sqrt{2\cdot 2}} = \frac{3\cdot\sqrt{10}}{2\cdot 2} = \frac{3\sqrt{10}}{4}$

An expression with exactly two terms is called a binomial expression. Two binomial expressions whose first terms are equal and last terms are opposite are called conjugates, i.e. $a+b$ and $a-b$ are conjugates.
If $a \geq 0$ and $b \geq 0$, then the binomials $x\sqrt{a}+y\sqrt{b}$ and $x\sqrt{a}-y\sqrt{b}$ are conjugates. We can use conjugates to rationalize denominators that contain radical expressions. For example, let us rationalize $\frac{1}{\sqrt{3}+\sqrt{2}}$, $\sqrt{3}-\sqrt{2}$ is the conjugate of $\sqrt{3}+\sqrt{2}$ Therefore, we multiply the numerator and the denominator by $\sqrt{3}-\sqrt{2}$ to rationalize the denominator

$$\frac{1}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{1\cdot(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})\cdot(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \frac{\sqrt{3}-\sqrt{2}}{1} = \sqrt{3}-\sqrt{2}$$

1.1.2 Examples

■ **Example 1.1** Rationalize the denominators.

- $\dfrac{\sqrt{5}}{3-2\sqrt{2}}$

- $\dfrac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}-1}$

- $\dfrac{\sqrt{6}+\sqrt{2}}{1-\sqrt{3}}$

- $\dfrac{\sqrt{6}+\sqrt{2}}{1-\sqrt{3}}$

■

■ **Example 1.2** Rationalize the denominator and simplify

$$\frac{3}{\sqrt{3}+1}-\frac{2}{1-\sqrt{2}}+\frac{3}{3\sqrt{3}}$$

■ **Example 1.3** (MATHCOUNTS) If $1 \le a \le 10$ and $1 \le b \le 36$, for how many ordered pairs of integers (a,b) is $\sqrt{a+\sqrt{b}}$ an integer? ■

■ **Example 1.4** (MATHCOUNTS) Let $p \ominus q = \sqrt{p^2-q^2}$, and let $p \oplus q = \sqrt{p^2+q^2}$. What is the value of $(3 \oplus 4) \oplus (20 \ominus 16)$? ■

■ **Example 1.5** When the letters of the alphabet are assigned their integer values $(A=1, B=2, C=3, \ldots, Z=26)$ the word-product for JULY is the product of the letter-values for each of the letters in JULY. When the square root of JULY's word-product is put into its simplest radical form of a $\sqrt{b}$, where b has no perfect-square factors greater than 1, what is the value of b? ■

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