

## **Square Roots**

## Rationalizing The Denominator

Changing the denominator of a fraction from an irrational number to a rational number is called rationalizing the denominator of the fraction. Tationalizing the denominator does not change the value of the original fraction. To rationalize the chominator, we multiply the numerator and denominator of the fraction by a suitable factor. For example, if the fraction is in the form  $\frac{\sqrt{a}}{\sqrt{b}}$ , we multiply both the numerator and the denominator by  $\sqrt{b}$ 

So,  $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{\sqrt{b \cdot b}} = \frac{\sqrt{ab}}{b}$ , Note that  $\frac{\sqrt{a}}{\sqrt{b}}$  have the same value: they are equivalent fractions.

- Look at some more examples:  $\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{3} \cdot 2}{\sqrt{2} \cdot 2} \cdot \frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$   $\frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \cdot \frac{3\sqrt{3}}{3} = \sqrt{3}$ , and  $\frac{3\sqrt{5}}{2\sqrt{2}} = \frac{3\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{10}}{2\sqrt{22}} = \frac{3\sqrt{10}}{2\cdot 2} = \frac{3\sqrt{10}}{2\cdot 2}$

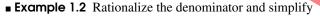
An expression with exclly terms is called a binomial expression. Two binomial expressions whose first terms are equal and last terms are opposite are called conjugates, i.e. a+b and a-b are

If  $a \ge 0$  and  $b \ge 0$ , then the binomials  $x\sqrt{a} + y\sqrt{b}$  and  $x\sqrt{a} - y\sqrt{b}$  are conjugates. We can use conjugates to rationalize denominators that contain radical expressions. For example, let us rationalize  $\frac{1}{\sqrt{3}+\sqrt{2}}$ ,  $\sqrt{3}-\sqrt{2}$  is the conjugate of  $\sqrt{3}+\sqrt{2}$  Therefore, we multiply the numerator and the denominator by  $\sqrt{3} - \sqrt{2}$  to rationalize the denominator

$$\tfrac{1}{\sqrt{3}+\sqrt{2}} \cdot \tfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \tfrac{1 \cdot (\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2}) \cdot (\sqrt{3}-\sqrt{2})} = \tfrac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \tfrac{\sqrt{3}-\sqrt{2}}{3-2} = \tfrac{\sqrt{3}-\sqrt{2}}{1} = \sqrt{3}-\sqrt{2}$$

## 1.1.2 Examples

- Example 1.1 Rationalize the denominators.
  - $\bullet \ \frac{\sqrt{5}}{3-2\sqrt{2}}$
  - $\bullet \ \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}-1}$
  - $\bullet \ \frac{\sqrt{6} + \sqrt{2}}{1 \sqrt{3}}$
  - $\bullet \ \frac{\sqrt{6} + \sqrt{2}}{1 \sqrt{3}}$



$$\frac{3}{\sqrt{3}+1} - \frac{2}{1-\sqrt{2}} - \frac{3}{3\sqrt{3}}$$

- **Example 1.3** (MATHCOUNTS) If  $1 < a \ge 10$  and  $1 \le b \le 36$ , for how many ordered pairs of integers (a,b) is  $\sqrt{a+\sqrt{b}}$  an integer
- **Example 1.4** (MATHCCLINGS) Let  $p \ominus q = \sqrt{p^2 q^2}$ , and let  $p \ominus q = \sqrt{p^2 + q^2}$ . What is the value of  $(3 \ominus 4) \ominus (20 \ominus 1)$ ?
- Example 1.5 When the letters of the alphabet are assigned their integer values (A = 1, B = 2, C = 3, ..., Z = 26) the word-product for JULY is the product of the letter-values for each of the letters in JULY. When the square root of JULY's word-product is put into its simplest radical form of a  $\sqrt{b}$ , where b has no perfect-square factors greater than 1, what is the value of b?