



1. Algebra

1.1 Special Factorization and Manipulations

1.1.1 Identities

Theorem 1.1.1 — Identities. Things to remember:

- $a^2 - b^2 = (a - b)(a + b)$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

1.1.2 Examples

- **Example 1.1** Let $x = 2001^{1002} - 2001^{-1002}$ and $y = 2001^{1002} + 2001^{-1002}$. Find $x^2 - y^2$. ■

- **Example 1.2** What is the value of $52,683 \times 52,683 - 52,660 \times 52,706$? (MATHCOUNTS) ■

- **Example 1.3** The difference of two positive integers is 34 and the product is 2015. What is the sum of the squares of them? ■

- **Example 1.4** Find $x^2 + \frac{1}{x^2}$ given that $x + \frac{1}{x} = 5$ ■

- **Example 1.5** Find $x^4 + \frac{1}{x^4}$ given that $x + \frac{1}{x} = 5$ ■

- **Example 1.6** Find $x^3 + \frac{1}{x^3}$ given that $x + \frac{1}{x} = 5$ ■

- **Example 1.7** A rectangle has perimeter 10 and diagonal $\sqrt{15}$. What is its area? ■