- 1. When the expression  $\cdot 5^{22}$  is multiplied out, how many digits does the number have?
- 2. Which of the numbers

$$2^{3^4}, 2^{4^3}, 3^{4^2}, 4^{3^2}, 4^{3^2}$$

has the greatest value?

3. Simplify the following expressions

• 
$$\frac{\sqrt{\sqrt{5}+2} - \sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}-1}}$$
  
•  $\frac{1+\sqrt{2}}{\sqrt{\sqrt{2}+\sqrt{11+\sqrt{72}}}}$ 

- 4. Given that  $\sqrt[4]{x^2 \sqrt[4]{x^2 \sqrt[4]{x^2 \dots}}} = 2$ , find *x*.
- 5. For what values of m does the equation  $x^2 + 3mx 5m 1 = 0$  have a double root?
- 6. Find the sum of all possible x values such that  $\frac{1}{x} + \frac{1}{y} = \frac{1}{7}$ , where x and y are positive integers.
- 7. The sum of nine consecutive integers is 216. What is the smallest of the nine integers?
- 8. The median of an arithmetic sequence of eight integers is 44. If the first term is 30, what is the value of the eighth term?
- 9. Consider numbers of the form 10n + 1, where n is a positive integer. We shall call such a number grime if it cannot be expressed as the product of two smaller numbers, possibly equal, both of which are of the form 10k + 1, where k is a positive integer. How many grime numbers are there in the sequence 11, 21, 31, 41, ..., 981, 991?
- 10. Peter has 25 cards, each printed with a different integer from 1 to 25. He wishes to place N cards in a single row so that the numbers on every adjacent pair of cards have a prime factor in common. What is the largest value of N for which this is possible?
- 11. How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?
- 12. How many three-digit numbers can be built from the digits in the list 2, 3, 5, 5, 5, 6, 6?
- 13. How many numbers can be expressed as a sum of four distinct members of the set  $\{17, 21, 25, 29, 33, 37, 41\}$ ?

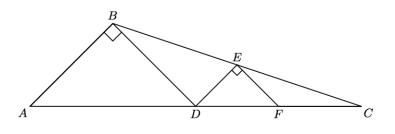
14. The dots in the grid below are equally spaced vertically and horizontally, with each dot 1 unit from its closest neighbors. How many different squares of any size can be formed by connecting four of the dots in the figure?



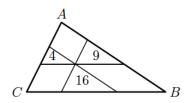
- 15. An urn contains marbles of four colors: red, white, blue, and green. When four marbles are drawn without replacement, the following events are equally likely:
  - (a) the selection of four red marbles;
  - (b) the selection of one white and three red marbles;
  - (c) the selection of one white, one blue, and two red marbles; and
  - (d) the selection of one marble of each color.

What is the smallest number of marbles that the urn could contain?

- 16. A triangle in the coordinate plane has vertices at (-4, 0), (6, 0) and (0, 5). The line  $y = \frac{5}{4}x + c$ , where c is a positive number, divides the triangle into a trapezoid and a smaller triangle whose areas, respectively, are in the ratio 5 : 4. What is the value of c?
- 17. Triangles ABD and DEF are isosceles right triangles. Points A, D, F and C are collinear, and points B, E and C are collinear. If AB = BD = 4 and ED = EF = 2, what is the length of segment AC?



- 18. In triangle ABC, H is on BC so that AH is an altitude, and E is on AC so that BE is an angle bisector. If  $\angle BEA = 45^{\circ}$ , then compute the degree measure of  $\angle EHC$ .
- 19. In  $\triangle ABC$ , line segments are drawn parallel to each of the sides dividing the triangle into six regions. The areas of three regions are shown in the figure. What is the total area of  $\triangle ABC$ ?



20. In the rectangle shown, the area of each circle is  $\pi(\sqrt{3}-1)$  square units. In square units, what is the area of the rectangle?

