1. Given that $\sqrt[3]{x+2} - \sqrt[3]{x-2} = 4$. Find x^3.

2. Evaluate $\sqrt{50 \cdot 51 \cdot 52 \cdot 53 + 1}$

3. a, b, c, and d are four consecutive even natural numbers. The sum of a and c is one fifth of the product of b and d. Find a, b, c, and d.

4. Let x_1 and $x_2$be the roots of the equation $kx^2 + (k-1)x - 2 + k = 0$. Given that $\dfrac{2}{x_1+3} + \dfrac{2}{x_2+3} = \dfrac{3}{2}$. Find k.

5. Compute $\dfrac{\lfloor \sqrt[4]{1} \rfloor \cdot \lfloor \sqrt[4]{3} \rfloor \cdot \lfloor \sqrt[4]{5} \rfloor \ldots \lfloor \sqrt[4]{2015} \rfloor}{\lfloor \sqrt[4]{2} \rfloor \cdot \lfloor \sqrt[4]{4} \rfloor \cdot \lfloor \sqrt[4]{6} \rfloor \ldots \lfloor \sqrt[4]{2016} \rfloor}$

6. Find the area of the region enclosed by the graph of $|x - 60| + |y| = |x/4|$.

7. Find all pairs of integers n and k, with $2 < k < n$, such that the binomial coefficients
$$\binom{n}{k-1}, \quad \binom{n}{k}, \quad \binom{n}{k+1}$$
form an increasing arithmetic sequence.

8. A sequence of positive integers with $a_1 = 1$ and $a_9 + a_{10} = 646$ is formed so that the first three terms are in geometric progression, the second, third, and fourth terms are in arithmetic progression, and, in general, for all $j \geq 1$, the terms a_{2j-1}, a_{2j}, a_{2j+1} are in geometric progression, and the terms a_{2j}, a_{2j+1}, and a_{2j+2} are in arithmetic progression. Find the greatest term in this sequence that is less than 1000.

9. The function f defined by
$$f(x) = \frac{ax+b}{cx+d},$$
where a, b, c, and d are nonzero real numbers, has the properties $f(19) = 19$, $f(97) = 97$, and $f(f(x)) = x$ for all values of x except $-d/c$. Find the unique number that is not in the range of f.

10. If x is a positive integer and
$$x(x+1)(x+2)(x+3) + 1 = 379^2,$$
compute x.

11. Find all values of x such that
$$4x^2 - 6x - 41 + \frac{1}{2x^2 - 3x - 19} = 0.$$

12. Let
$$P(x) = x^4 + ax^3 + bx^2 + cx + d,$$
where a, b, c, and d are constants. If $P(1) = 10$, $P(2) = 20$, and $P(3) = 30$, compute
$$\frac{P(12) + P(-8)}{10}.$$

13. Let r_1, r_2, r_3 be the 3 zeroes of the cubic polynomial $x^3 - x - 1 = 0$. Then, the expression
$$r_1(r_2 - r_3)^2 + r_2(r_3 - r_1)^2 + r_3(r_1 - r_2)^2$$

14. Compute the number of permutations $x_1 \dots x_6$ of the integers $1, \dots, 6$ such that $x_{i+1} \le 2x_i$ for all i, $1 \le i \le 6$.

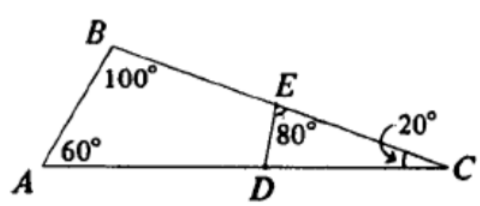
15. An n-sided die has the integers between 1 and n (inclusive) on its faces. All values on the faces of the die are equally likely to be rolled. An 8-sided die, a 12-sided die, and a 20-sided die are rolled. Compute the probability that one of the values rolled is equal to the sum of the other two values rolled.

16. Compute the largest of the three prime divisors of $13^4 + 16^5 - 172^2$

17. A paper equilateral triangle ABC has side length 12. The paper triangle is folded so that vertex A touches a point on side $\overline{BC}$ a distance 9 from point B. The length of the line segment along which the triangle is folded can be written as $\frac{m\sqrt{p}}{n}$, where m, n, and p are positive integers, m and n are relatively prime, and p is not divisible by the square of any prime. Find $m + n + p$.

18. In a circle, parallel chords of lengths 2, 3, and 4 determine central angles of α, β, and $\alpha + \beta$ radians, respectively, where $\alpha + \beta < \pi$. If $\cos\alpha$, which is a positive rational number, is expressed as a fraction in lowest terms, what is the sum of its numerator and denominator?

19. In $\triangle ABC$, E is the midpoint of side BC and D is on side AC. If the length of AC is 1 and $\angle BAC = 60°, \angle ABC = 100°, \angle ACB = 20°$ and $\angle DEC = 80°$, then the area of $\triangle ABC$ plus twice the area of $\triangle CDE$ equals



20. Let $ABCD$ be a trapezoid with $AB \parallel CD$. The bisectors of $\angle CDA$ and $\angle DAB$ meet at E, the bisectors of $\angle ABC$ and $\angle BCD$ meet at F, the bisectors of $\angle BCD$ and $\angle CDA$ meet at G, and the bisectors of $\angle DAB$ and $\angle ABC$ meet at H. Quadrilaterals $EABF$ and $EDCF$ have areas 24 and 36, respectively, and triangle ABH has area 25. Find the area of triangle CDG.

21. What is the area of a trapezoid whose altitude has a length of 12 and one of whose perpendicular diagonals has a length of 15?

22. Three circles with radii $r_1 < r_2 < r_3$ are placed so that they are tangent to a horizontal line, and so that adjacent circles are externally tangent to each other. F, G, H, I, J, and K are the points of tangency of the circles to the horizontal line, as shown. The lengths of FG, HI ,JK , in no particular order, are 18, 20 and 22. Determine, with justification, the values of r_1, r_2 and r_3.